**MATRIX FACTORIZATION**

We find the solution of Bx=b simply by calculating x=B-1b, but sometimes calculating B-1 is cumbersome or when B is nearly singular. So, we use factorization or decomposition methods to break B into easy to use matrices. Some of the factorisation methods are discussed below…

**LU and PLU Factorization for basis B**

­­­It’s simply the gaussian elimination method that we have learnt already in 12th class and also in mth102. We convert B matrix to upper triangular matrix by performing series of operations. We multiply a series of matrix to obtain the same. Name those matrices as Gi and we do permutations Pi after every operation.

(Gr Pr ). . . (G2 P2 )(G1 P1)B = U

Multiplying this matrices on either side of the equation Bx=b will give

Ux = b’

Where b’=(Gr Pr ). . . (G2 P2 )(G1 P1)b

If no permutations are performed, Gr ….Gi is lower triangular, and denoting its (lower triangular) inverse as L, we have the factored form B = LU for B, hence its name.

Also, if PT is a permutation matrix that is used to a priori rearrange the rows of B and we then apply the Gaussian triangularization operation to derive L-1PTB = U, we can write B = (PT)-1LU = PLU, noting that PT = P-1. Hence, this factorization is sometimes called a PLU decomposition.

**QR and QRP Factorization for a Basis B**

This factorization is most suitable and is used frequently for solving dense equation systems. Here the matrix B is reduced to an upper triangular form R by premultiplying it with a sequence of square, symmetric orthogonal matrices Qi.

Bi-1 = Qi-1 ....Q1B

The matrix Qi is a square, symmetric orthogonal matrix of the form Qi = 1 – yi qi qiT, where qi = (0, ..., 0, qii ,...,qni)' and yi E RT are suitably chosen to perform the foregoing operation.

Defining Q = Qn-1…...Q1 , we see that Q is also a symmetric orthogonal matrix and that QB = R, or that B = QR, since Q = QT = Q-1 that is, Q is an involutory matrix.

Now, to solve Bx = b, we equivalently solve QRx = b or Rx = Qb by finding b’ = Qb first and then solving the upper triangular system Rx = b’ via back-substitution. Note that since ||Qv|| = ||v|| for any vector v, we have ||R|| = ||QR|| = ||B||, so that R preserves the relative magnitudes of the elements in B, maintaining stability. This is its principal advantage.

**Cholesky Factorization LLT and LDLT for Symmetric, Positive Definite Matrices B**

This method is only applicable for square symmetric matrix B. We will represent this matrix as B=L\*LT, where L is a Lower-triangular matrix. So that L\*LT becomes a symmetric matrix and then by comparing the elements of both the symmetric matrices, we could figure out the elements of L.

The equation system Bx=b, can now be solved via LT(Lx)=b, through the solution of two triangular systems of equations. We first find y to satisfy Ly = b and then compute x via the system LTx = y.

Sometimes we write B = LDLT for the sake of accuracy as it avoids calculating the square root which otherwise was to be calculated in finding the elements of B.